Miniworkshop
Modern Valuation Theory

Jena, 19–21 July 2023

Contents

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https://github.com/maximelucas/AMCOS_booklet

Schedule

Wednesday, July 19

9:00–9:30	**Registration**	
9:30–10:30	**Andreas Bernig** Frankfurt	The Weyl principle in pseudo-Riemannian geometry
10:30–11:00	**Coffee**	
11:00–11:30	**Semyon Alesker** Tel Aviv	Non-Archimedean analogues of the space of valuations on convex sets
11:30–12:00	**Juan Andrés Trillo** Barcelona	Tube formulas for valuations in complex space forms
12:00–14:00	**Lunch**	
14:00–15:00	**Dmitry Faifman** Tel Aviv	Some Whitney-type extension properties of valuations
15:00–15:30	**Coffee**	
15:30-16:00	**Jakob Schuhmacher** Jena	Continuous Translation-Invariant Curvature Measures
16:00-16:30	**Oscar Ortega Moreno** Vienna	Iterations of Minkowski Valuations

Thursday, July 20

9:30–10:30	**Monika Ludwig** Vienna	Valuations on Convex Functions
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11:00–11:30	**Jonas Knörr** Vienna	Algebraic Integral Geometry for Monge-Ampère operators
11:30-12:00	**Leo Brauner** Vienna	Fixed points of Mean Section Operators
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14:00–15:00	**María A. Hernández Cifre** Murcia	Dual volumes and dual Steiner polynomials
15:00–15:30	**Coffee**	

Friday, July 21

9:30–10:30	**Benjamin Schröter** Frankfurt	Valuations in matroid theory
10:30–11:00	**Coffee**	
11:00–11:30	**Ansgar Freyer** Vienna	Tensor Valuations on Lattice Polygons Beyond Ehrhart Coefficients
11:30-12:00	**Simon Ellmeyer** Vienna	Complex L_p-Intersection Bodies

List of Abstracts

The Weyl principle in pseudo-Riemannian geometry

Andreas Bernig*, *Goethe University Frankfurt We 9:30–10:30

The classical Weyl principle states that the volume of a tube around a compact submanifold of euclidean space is a polynomial in the radius. The (suitably normalized) coefficients depend only on the intrinsic geometry of the manifold and not on the embedding. They are called intrinsic volumes and comprise as special cases the volume, the total scalar curvature, and the Euler characteristic. Using Alesker's theory of valuations on manifolds we construct intrinsic volumes of pseudo-Riemannian manifolds that satisfy a version of Weyl's principle. The intrinsic volumes can be extended to certain manifolds with a sign-changing metric and satisfy a version of the Chern-Gauss-Bonnet theorem. This is joint work with D. Faifman and G. Solanes.

Non-Archimedean analogues of the space of valuations on convex sets

Semyon Alesker, Tel Aviv University We 11:00–11:30

In the last two decades there were discovered a number of algebraic operations on the classical space of translation invariant of valuations on convex sets: product, convolution, Fourier type transform, pull-back, and push-forward. In this talk we introduce an analogues of the space of such even valuations over any local (e.g. p-adic) field which carries similar structures and satisfies Poincare duality and hard Lefschetz theorem.

Tube formulas for valuations in complex space forms

Juan Andrés Trillo*, *Autonomous University of Barcelona We 11:30–12:00

Given an isometry invariant valuation on a complex space form we compute its value on the tubes of sufficiently small radii around a set of positive reach. This generalizes classical formulas of Weyl, Gray and others about the volume of tubes. We also develop a general framework on tube formulas for valuations in riemannian manifolds. This is joint work with Gil Solanes.

Some Whitney-type extension properties of valuations

Dmitry Faifman, Tel Aviv University online, We 14:00–15:00

Assume that a collection of valuations is given on a family of subspaces in a linear space V. Are they the restrictions of a single valuation on V? Clearly, compatibility of the given data on intersections is a necessary condition. Is it sufficient? This general question encases several distinct problems, including a Nash-type embedding theorem for smooth valuations on manifolds, and a sheaf-type property of translation-invariant valuations. We will present some results in this direction, and explain some aspects of their proofs. Based on a joint work with Georg Hofstätter.

Continuous Translation-Invariant Curvature Measures

Jakob Schuhmacher, Friedrich Schiller University Jena We 15:30–16:00

We introduce the concept of continuous translation-invariant curvature measures on a real finite dimensional vector space, which generalizes the notion of smooth translation-invariant curvature measures due to Bernig and Fu. The set of these curvature measures forms a vector space, denoted by Curv. We will see that Curv admits a grading analogous to the McMullen decomposition for valuations and we equip Curv with a Banach space topology. Making use of an embedding into a space of vector-valued valuations, we will give explicit descriptions of certain special classes of curvature measures. This is joint work in progress with T. Wannerer.

Iterations of Minkowski Valuations

Oscar Ortega Moreno, TU Wien We 16:00–16:30

In this talk, we show that for any sufficiently regular even Minkowski valuation Φ which is homogeneous and intertwines rigid motions, and for any convex body K in a smooth neighborhood of the unit ball, there exists a sequence of positive numbers $(\gamma_m)_{m=1}^{\infty}$ such that $(\gamma_m \Phi K)_{m=1}^{\infty}$ converges to the unit ball with respect to the Hausdorff metric.

Valuations on Convex Functions

Monika Ludwig, TU Wien Th 9:30–10:30

Let $\mathrm{Conv}(\mathbb{R}^n)$ be the space of proper, lower semicontinuous, convex functions $v : \mathbb{R}^n \to (-\infty, \infty]$ and $\mathbb{A}$ an Abelian semi-group. A functional $\mathrm{Z}\colon \mathrm{Conv}(\mathbb{R}^n) \to \mathbb{A}$ is called a *valuation* if

$$\mathrm{Z}(f \vee g) + \mathrm{Z}(f \wedge g) = \mathrm{Z}(f) + \mathrm{Z}(g)$$

for all $f, g \in \mathrm{Conv}(\mathbb{R}^n)$ such that the pointwise maximum $f \vee g$ and the pointwise minimum $f \wedge g$ are in $\mathrm{Conv}(\mathbb{R}^n)$. We present an overview of results on valuations on the space of convex functions on $\mathbb{R}^n$. In particular, classification results of real and measure-valued valuations on convex functions on $\mathbb{R}^n$ will be described.

Algebraic Integral Geometry for Monge-Ampère operators

Jonas Knörr, TU Wien Th 11:00-11:30

Monge-Ampère operators play an important role in the construction of invariant valuations on convex functions. Conversely, translation equivariant Monge-Ampère operators can be identified with certain measure-valued valuations on the space of finite convex functions. In this talk we will discuss how this description gives rise to additive kinematic formulas encoded in a convolution product. Moreover, we will see that this product structure satisfies the mixed Hodge-Riemann bilinear relations, which provides a natural way to interpret the higher order versions of Alexandrov's mixed discriminant inequality.

Fixed points of Mean Section Operators

Leo Brauner, TU Wien Th 11:30–12:00

Several geometric inequalities for Minkowski valuations can be reduced to the determination of fixed points. In this talk, we prove that for a large class of Minkowski valuations, including the Mean Section Operators, there is a neighborhood of the unit ball where the only fixed points are Euclidean balls. Our approach unifies and extends previous results by Ivaki, Schuster, and Ortega-Moreno.

This is joint work with Oscar Ortega-Moreno.

Dual volumes and dual Steiner polynomials

María A. Hernández Cifre, University of Murcia online, Th 14:00–15:00

Dual (mixed) volumes and dual Steiner polynomials are central notions in the well-known dual Brunn-Minkowski theory. Many interesting questions and generalizations arise around these two concepts, and our aim in this talk is to present some of them: how to characterize dual Steiner polynomials, how they behave, convex bodies that are determined by their dual volumes, or possible extensions to other functionals or close related polynomials. The contrast of these results with the corresponding ones in the classical Brunn-Minkowski setting will be also shown.

Valuations in matroid theory

Benjamin Schröter, Goethe University Frankfurt Fr 9:30–10:30

Matroids are a combinatorial abstraction of both graphs and vector configurations that appear all over in mathematics and have a rich combinatorial structure. One may associates a matroid with its base polytope. This geometric point of view reveals that many important matroid invariants are valuations. These invariants include Tutte polynomials and their specializations as well as Hilbert–Poincaré series of the Chow rings of geometries and many more.

In this talk I will give a brief introduction to matroids and present a framework that allows us to compute matroid invariants on large classes of matroids, e.g., sparse paving and elementary split matroids, explicitly. The concept of split matroids introduced by Joswig and myself is relatively new. However, this class appears naturally in this context. Moreover, this class contains the intensively studied class of (sparse) paving matroids. I will demonstrate the framework by looking at several examples.

This talk is based on the preprint 'Valuative invariants for large classes of matroids' which is joint work with Luis Ferroni.

Tensor Valuations on Lattice Polygons Beyond Ehrhart Coefficients

Ansgar Freyer, TU Wien Fr 11:00–11:30

In 2017, Ludwig and Silverstein showed that any unimodular equivariant and translation covariant tensor valuation of lattice polytopes of rank at most 8 is a linear combination of the so-called Ehrhart tensor coefficients. Moreover, they constructed a valuation of rank 9 which does not arise in this way.

The talk addresses the problem of classifying the missing tensor valuations for higher ranks in the plane. Using the representation theory of finite groups, we give a complete classification of the 1-homogeneous valuations in this setting. As a byproduct we obtain a new measure of asymmetry of convex lattice polygons. Moreover, we present partial results on the valuations of higher homogeneity degree. This is work in progress with Monika Ludwig and Martin Rubey.

Complex L_p-Intersection Bodies

Simon Ellmeyer, TU Wien Fr 11:30–12:00

Interpolating between the classical notions of intersection and polar centroid bodies, (real) L_p-intersection bodies, for $-1 < p < 1$, play an important role in the dual L_p-Brunn–Minkowski theory. Inspired by the recent construction of complex centroid bodies, a complex version of L_p-intersection bodies, with range extended to $p > -2$, is introduced, interpolating between polar complex centroid and complex intersection bodies. This is joint work with Georg C. Hofstätter.

List of Participants

Semyon Alesker	Tel Aviv University
Andreas Bernig	Goethe University Frankfurt
Marie-Charlotte Brandenburg	MPI Leipzig
Leo Brauner	TU Wien
Simon Ellmeyer	TU Wien
Dmitry Faifman	Tel Aviv University
Ansgar Freyer	TU Wien
María A. Hernández Cifre	University of Murcia
Georg Hofstätter	Friedrich Schiller University Jena
Christian Kipp	TU Berlin
Jonas Knörr	TU Wien
Jan Kotrbatý	Goethe University Frankfurt
Monika Ludwig	TU Wien
Fernanda Helen Moreira Baeta	TU Wien
Oscar Ortega Moreno	TU Wien
Christian Richter	Friedrich Schiller University Jena
Benjamin Schröter	Goethe University Frankfurt
Jakob Schuhmacher	Friedrich Schiller University Jena
Juan Andrés Trillo	Autonomous University of Barcelona
Thomas Wannerer	Friedrich Schiller University Jena

Useful Information

Talks will be held at the **lecture hall (room 024)**, which is located on the ground floor of the old university building of Friedrich Schiller University in Fürstengraben 1, Jena, Germany. It can be accessed either from the main entrance at Fürstengraben 1 or from the entrance at Schloßgasse.

Coffee will be served in the foyer next to the lecture hall.

Wi-Fi will be available during the conference by the eduroam network.

The **conference dinner** will take place on Thursday from 19:00 at the restaurant "Zur Noll", at Oberlauengasse 19, Jena (about 150m from the conference venue).

More details can be found at the **conference website**

`https://modernval2023.uni-jena.de/`

Organizing Committee

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